Reduction Formula 104

(xi) Form $\int\limits_{0} \cos^m x \sin nx \, dx$

$$\Rightarrow \boxed{I_{m,n} = -\frac{\cos^m x \cos nx}{m+n} + \frac{m}{m+n} I_{m-1,n-1}}$$

(xii) Form $\int\limits_{0}^{\pi/2} \cos^m x \sin nx \, dx$

$$\boxed{I_{m,n} = \frac{1}{m+n} + \frac{m}{m+n} I_{m-1,n-1}}$$

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(xiii) Form $\int\limits_{0}^{\infty} e^{-ax} \cos^n x \, dx$

$$\Rightarrow \boxed{I_{m,n} = \frac{a}{n^2+a^2} + \frac{n(n-1)}{n^2+a^2} I_{n-2}}$$

Ques Prove $\int\limits_{0}^{\pi/2} \cos^{n-2} x \sin nx \, dx = \frac{1}{n-1}$ for $n > 1$

Soln $I_{m,n} = \int\limits_{0}^{\pi/2} \cos^{n-2} x \sin nx \, dx$

where $m = n-2$

$$\therefore I_{n-2,n} = \frac{1}{n-2+n} + \frac{n-2}{n-2+n} I_{n-3,n-1}$$

$$= \frac{1}{2(n-1)} + \frac{n-2}{2(n-1)} I_{n-3,n-1} \quad \text{--- (1)}$$

$$I_{n-3,n-1} = \frac{1}{n-3+n-1} + \frac{n-3}{n-3+n-1} I_{n-4,n-2}$$

$$= \frac{1}{2(n-2)} + \frac{n-3}{2(n-2)} I_{n-4,n-2}$$

$$\therefore (1) \Rightarrow I_{n-2,n} = \frac{1}{2(n-1)} + \frac{\cancel{(n-2)}}{2(n-1)} \cdot \frac{1}{2\cancel{(n-2)}} + \frac{\cancel{(n-2)}(n-3)}{2(n-1)2\cancel{(n-2)}} \cdot I_{n-4,n-2}$$

$$= \frac{1}{2(n-1)} + \frac{1}{2^2(n-1)} + \frac{n-3}{2^2(n-1)} I_{n-4,n-2}$$

$$= \frac{1}{2(n-1)} + \frac{1}{2^2(n-1)} + \frac{n-3}{2^2(n-1)} \left[\frac{1}{n-4+n-2} + \frac{n-4}{n-4+n-2} I_{n-5,n-3} \right]$$

$$= \frac{1}{2(n-1)} + \frac{1}{2^2(n-1)} + \frac{\cancel{(n-3)}}{2^2(n-1)2\cancel{(n-3)}} + \frac{\cancel{(n-3)}(n-4)}{2^2(n-1)2\cancel{(n-3)}} I_{n-5,n-3}$$

$$= \frac{1}{2(n-1)} + \frac{1}{2^2(n-1)} + \frac{1}{2^3(n-1)} + \frac{(n-4)}{2^3(n-1)} I_{n-5,n-3}$$

Similarly proceeding $(n-5)$ times we get

$$\Rightarrow I_{n-2,n} = \frac{1}{2(n-1)} + \frac{1}{2^2(n-1)} + \frac{1}{2^3(n-1)} + \cdots + \frac{1}{2^{n-2}(n-1)} + \frac{n-(n-1)}{2^{n-2}(n-2)} I_{0,2}$$

$$= \frac{1}{2(n-1)} + \frac{1}{2^2(n-1)} + \frac{1}{2^3(n-1)} + \cdots + \frac{1}{2^{n-2}(n-1)} + \frac{1}{2^{n-2}(n-2)} I_{0,2}$$

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$$= \frac{1}{n-1}\left[\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^{n-2}} + \frac{1}{2^{n-2}}\int_0^{\pi/2} \sin 2x\,dx\right]$$

$$\Rightarrow I_{n-2,n} = \frac{1}{n-1}\left[\left\{\frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{n-2}}\right\} + \frac{1}{2^{n-2}}\left[-\frac{\cos 2x}{2}\right]_0^{\pi/2}\right]$$

$$= \frac{1}{n-1}\left[\frac{\frac{1}{2}\left(1-\frac{1}{2^{n-2}}\right)}{1-\frac{1}{2}} + \frac{1}{2^{n-2}} \times [-0+1]\right]$$

$$= \frac{1}{n-1}\left[1 - \cancel{\frac{1}{2^{n-2}}} + \cancel{\frac{1}{2^{n-2}}}\right] = \frac{1}{n-1}, \text{ if } n>1$$

$$\Rightarrow \int_0^{\pi/2} \cos^{n-2}x \sin nx\,dx = \frac{1}{n-1}, \text{ if } n>1$$

Proved